

A Modification of Pedersen's Model for Saturated Crude Oil Viscosities Using Standard Black Oil PVT Data

by

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Abstract

This paper presents a modification of Pedersen's corresponding states compositional viscosity model that enables viscosity prediction for black oil systems when there are no compositional data available. This model can be easily implemented in any reservoir simulation software, it can be easily tuned, and it provides better estimates of oil viscosity than the existing correlations.

Viscosity from 324 sets of differential liberation data consisting of 2343 observations covering a wide range of pressure, temperature, and oil density were used to develop the correlation which retains most of the functional form of Pedersen's model. These modifications include (1) use of n-decane as the reference fluid, (2) consider the oil mixture as a single pseudocomponent with molecular weight and critical properties correlated to its density, and (3) add a functional dependence to solution gas/oil ratio and gas-specific gravity. The average error over 2343 viscosity observations was 0.9%. The model was tested against a second data set consisting of 150 observations and the average error was 0.7 %.

The predictions were compared with those predicted from the correlations of Khan *et al.* and of Petrosky that are applicable to the experimental conditions of our data sets. These average errors for these correlations were -28 % and 4.9 % respectively for the first data set; and -60.8 % and -1.4 % for the second data set.

Keywords: crude oil viscosity, corresponding states, method of calculation

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Introduction

Crude oil viscosity is important in the calculation of two-phase flow, gas-liquid flowing pressure traverses, tubing-string design, gas-lift design, and pipeline design. Most important of all, it is needed to calculate the recovery of the oil either from natural depletion or from recovery techniques such as waterflooding and gas-injection processes.

Live oil viscosity is a strong function of pressure, temperature, oil gravity, gas gravity, gas solubility, molecular sizes, and composition of the oil mixture. The variation of viscosity with molecular structure is not well known due to the complexity of crude oil systems. However, paraffin hydrocarbons do exhibit a regular increase in viscosity as the size and complexity of molecules increases.

Crude oil viscosity correlations are usually developed for three situations: above the bubble point pressure, at and below the bubble point pressure, and for dead oil. Dead oil is oil without gas in solution at atmospheric pressure. Above the bubble point, the composition of the oil mixture is constant and the viscosity changes mainly result from compressibility: the fluid becomes heavier and its viscosity increases. At some point during production, the pressure drops below the bubble point, gas comes out of solution, and the oil composition changes continuously. The oil becomes heavier and more viscous, and two phases will flow in the reservoir. This production path is simulated in the laboratory by a stage-wise flashing of the live oil at reservoir temperature. During this process the oil volumes, the amount of gas in solution, and the oil viscosity are determined. Fig. 1 shows a schematic of the differential liberation process that ends at atmospheric pressure, leaving a dead-oil.

Viscosity Correlations

Numerous viscosity-correlation methods have been proposed. None, however, has been used as a standard method in the oil industry. Since the crude oil composition is complex and often undefined, many viscosity estimation methods are geographically dependent. Most correlation methods can be categorized either as a black-oil type or as compositional.

Black-oil type correlations predict viscosities from available field-measured variables by fitting of an empirical equation. The correlating variables traditionally include a combination of solution gas/oil

ratios (R_s), bubble point pressure, oil API gravity, temperature, specific gas gravity, and/or the dead oil viscosity. Chew and Connally [1], Beggs and Robinson [2], Khan *et al.* [3], Kartoatmodjo and Smith [4] and Petrosky [5] correlated oil viscosity with temperature, pressure, oil gravity and solution gas/oil ratio.

The second method derives mostly from the principle of corresponding states and its extensions. Lohrenz *et al.*, [6] Ely and Hanley, [7] Pedersen *et al.* [8], Pedersen and Fredenslund [9] and Monnery *et al.* [10] are among the researchers following this trend. Lohrenz *et al.* [6] and Pedersen *et al.* [9] are probably the most common methods implemented in the majority of the commercial compositional reservoir simulators.

Corresponding states methods

Methods based upon the corresponding states theory predict the crude-oil viscosity as a function of temperature, pressure, composition of the mixture, mixture pseudocritical properties, and the viscosity of a reference substance evaluated at a reference pressure and temperature. A group of substances obeys the corresponding states principle with respect to viscosity if the functional dependence of the reduced viscosity (μ) on, e.g. the reduced temperature (T_r) and the reduced pressure (P_r) is the same for all substances within the group. Therefore, viscosity data are needed for only one of the components of the group, which is used as a reference fluid.

Lohrenz, Bray and Clark [6] published the now well-known LBC correlation suitable for gases and light oils. The LBC correlation is a fourth-degree polynomial in the reduced density of the mixture and that makes it very sensitive to this variable.

Ely and Hanley [7] published a model based on an extended corresponding-states principle for predicting the viscosity of non-polar pure fluids and their mixtures. The simple corresponding-states principle is closely obeyed by monatomic molecules but not by polyatomic fluids at high density. Pedersen *et al.* [9] introduced a third parameter (α) to correct for this deviation from the conventional corresponding states principle. This term accounts for the molecular size and density effects on viscosity. Their model eliminates the iterative procedure in Ely and Hanley [7] and performs a direct calculation of the viscosity.

Model Development

Since most of the features from our correlation resemble Pedersen *et al.* [9] model we rewrite their model here.

$$\mu_m(P, T) = \left(\frac{T_{cm}}{T_{co}} \right)^{\alpha_1} \left(\frac{P_{cm}}{P_{co}} \right)^{\alpha_2} \left(\frac{MW_m}{MW_o} \right)^{\alpha_3} \left(\frac{\alpha_m}{\alpha_o} \right) \mu_o(P_o, T_o), \quad (1)$$

where the coefficients \mathbf{a}_1 , \mathbf{a}_2 and \mathbf{a}_3 in Pedersen's model are -1/6, 2/3 and 1/2 respectively.

$$\alpha_m = 1.000 + 7.378 \times 10^{-3} \rho_{ro}^{1.847} MW_m^{0.5173} \quad (2)$$

$$\alpha_o = 1.000 + 0.031 \rho_{ro}^{1.847} \quad (3)$$

Here, \mathbf{r}_{ro} is the reduced density of the reference fluid. Pedersen *et al.* [9] used methane as the reference fluid. They used a BWR-equation in the form suggested by McCarty [11] to evaluate the density of methane. This density is evaluated at a reference pressure and temperature as indicated in Eq. (4)

$$\rho_{ro} = \frac{\rho_o \left(\frac{PP_{co}}{P_{cm}}, \frac{TT_{co}}{T_{cm}} \right)}{\rho_{co}} \quad (4)$$

while the pressure and temperature at which the reference viscosity (\mathbf{m}) is evaluated are,

$$P_o = \frac{PP_{co} \alpha_o}{P_{cm} \alpha_m} \quad \text{and} \quad T_o = \frac{TT_{co} \alpha_o}{T_{cm} \alpha_m} \quad (5)$$

The critical temperature and pressure are found using the mixing rules suggested by Mo and Gubbins [12] using the composition of the oil mixture. The method is highly sensitive to the characterization of the heavy fraction, usually known as the C_7^+ fraction. Our objective was to extend this model to black oil mixtures for which we do not have compositional information.

The limitation of methane as the reference substance is that when the reduced temperature of methane is below 0.4, it will freeze. This is above the reduced temperatures for most reservoir fluids. Pedersen *et al.* [9] solved this problem by modifying the viscosity model of Hanley *et al.* [13], while Monnery *et al.* [10] suggested using propane as a reference fluid .

To use Eq. (1) we needed to find simplified expressions for the molecular weight (MW_m), critical temperature and pressure (T_{cm} , and P_{cm}) of the mixture, and for the density and viscosity of the reference fluid. We initially used methane as the reference fluid, but rather than implementing Pedersen's modifications, which are tedious and add additional complexity to the model, we decided to use an alternative reference fluid. We selected n-decane for this purpose.

The viscosity and density data for n-decane were taken from various sources reported by Geopetrole [14] covering pressures from 14.7 psia to 7325 psia and temperatures from 492°F to 762°F. The density and viscosity of n-decane were fitted as a function of P and T using a stepwise regression procedure and the statistical software SAS [15]. The density, in lb/ft³, is

$$\rho_{C_{10}} = -1847.7998 \times T^{-1} + 168.1906 \times T^{-1/2} + 1.5043 \times 10^{-8} TP. \quad (6)$$

while the viscosity, in cp, is given by

$$\mu_{C_{10}} = 50992 \frac{1}{T} + 2321.5418 \times T^{-1/3} - 8775.2881 \times T^{-1/2} + 0.4775 \frac{P}{T} - 0.001272 \times P - 6.7057 \times 10^{-9} \times T^3 + 8.87 \times 10^{-7} \times PT. \quad (7)$$

The correlation coefficient for Eq. (6) is $R^2 = 0.9996$ with minimum and maximum errors of -1.47% and $+1.82\%$ respectively. Eq. (7) has a correlation coefficient of $R^2 = 0.9998$ and gives minimum and maximum errors of -3.11% and $+8.21\%$ respectively. The pressures and temperatures in Eqs. (6) and (7) are in psia and degrees R respectively.

The specific gravity of the oil was evaluated from a material balance using the reported values of formation volume factor (B_o), solution gas/oil ratio (R_s), and gas specific gravity according to MacCain [16]. The reported specific gravity of the gas was for the separator at 100 psi rather than

at atmospheric pressure, however; the error introduced in the determination of specific gravity of the oil is negligible.

The oil mixture was lumped into a single pseudocomponent for which the critical temperature, the critical pressure, and the molecular weight were correlated to the oil specific gravity.

Most correlations for the critical properties require at least two properties from the molecular weight, the density, and the normal boiling point. We had only one of these variables. To overcome this problem we assumed that for most oils the percentage of paraffinic compounds dominates and in that case we correlated the normal boiling versus specific gravity of oil at reservoir conditions ($g_{o,R}$). Once this was determined the molecular weight was correlated to the normal boiling point in R. The data to develop these correlations were reported by Ahmed [17] and Whitson [18].

The normal boiling point in R, and the mixture molecular weight are given by:

$$T_b = 3540.53 - 385.934312 \frac{1}{\gamma_{o,R}} - 5431.82548 \times \gamma_{o,R} + 4193.44761 \times \gamma_{o,R}^3 \quad (8)$$

$$MW_m = 64.611 \times \exp(0.0022 \times T_b) \quad (9)$$

Once these two properties were obtained the critical pressure P_{cm} was obtained using the Riazi-Daubert [19] correlation, while the T_{cm} was calculated using the following relationship:

$$T_{cm} = 24.2787 \times T_b^{0.58848} \times g_{o,R}^{0.3596} \quad (10)$$

We observed that the critical pressure, P_{cm} , was not always monotonic as the oil became heavier. Particularly for lighter oils, P_{cm} went through a maximum and it decreased at the later stages of depletion. Since we wanted to generalize the equation for heavier and lighter oils, we selected V_{cm} as the correlating variable since it increases monotonically as the oil becomes heavier. The correlation used for V_{cm} was also from Riazi-Daubert [19].

If the hydrocarbon mixture had a larger percentage of aromatic compounds, the correlation for the molecular weight and normal boiling points would have to be modified. For example, the

molecular weight of an aromatic component with a T_b of 640°F is about 179 lb/lb-mol, while the same boiling point corresponds to a paraffinic mixture with average molecular weight of about 260 lb/lb-mol.

The data base was screened for consistency following an automated scheme developed by Dexheimer and Barrufet [20]. The method screens for outliers in a given data set and discards the viscosity points that do not follow a consistent pattern, i.e. viscosity should increase monotonically as the pressure decreases.

The final model for the oil viscosity was

$$\mu_m(P, T) = \left(\frac{T \times T_{cm}}{T_{cC10}^2} \right)^{1.0286} \left(\frac{V_{cm}}{V_{cC10}} \right)^{0.9841} \left(\frac{R_s \times \gamma_{o,R}}{R_{sb}} \right)^{-0.1362\gamma_{o,R}} \left(\frac{MW_m}{MW_{C10}} \right)^{-3.9243} R_{sb}^{-0.4471} API^{-2.2902} \quad (11)$$

$$\times \exp \left(-0.2606 \times \left(\frac{B_o}{B_{ob}} \right)^3 - 0.02359 \frac{P \times V_{cm}}{T_{C10}} + 2.1388 \frac{\rho_{C10}}{\rho_{cC10}} + 0.1930 \mu_{C10}(P_o, T_o) \right)$$

where B_o , the formation volume factor, is dimensionless, $\gamma_{g,100}$ is the specific gravity of gas at 100 psia, $\gamma_{o,R}$ is the specific gravity of the oil at standard conditions, R_s is the solution gas/oil ratio in SCF/STB (standard cubic feet per stock tank barrel). R_{sb} and B_{ob} are evaluated at the bubble point pressure.

The advantage of this model is that it can be easily retuned if necessary using linear regression. The exponent for the variable (B_o/B_{ob}) was determined independently and it is left as a fixed parameter. The n-decane density and viscosity were evaluated at the same reference pressure and temperature indicated in Eqs. (4) and (5), and the same values for \mathbf{a}_m and \mathbf{a}_b defined in Eqs. (2) and (3) were used. No attempt was made to retune these values.

Results

Our model was developed using a data set of 2343 points (Data Set 1) and it was validated with an independent data set from Core laboratories consisting of 150 observations (Data Set 2). Table 1 indicates the ranges of viscosity, temperature, and pressure for the two sets.

To evaluate the performance of this model we selected two models which do not assume the knowledge of the dead-oil viscosity. Khan *et al.*[3] proposed a correlation for the bubble point viscosity, while Petrosky [5] proposed a correlation for the dead-oil viscosity. The experimental ranges of pressure, oil gravity, temperature, and solution-gas/oil ratios are similar to those of our data bases.

Fig. 2 shows the performance of the adapted and untuned Pedersen model Eq. (1) with the original coefficients but using n-decane as the reference fluid. Figs. 3 to 5 show predicted versus experimental viscosities for Data Set 1 according to Khan's *et al.* correlation, Petrosky's correlation, and this work respectively. Figs. 6 to 9 show the predicted versus experimental viscosities for Data Set 2 according to the untuned Pedersen's mode Eq. 1, Khan's *et al.* correlation, Petrosky's correlation, and this work respectively

If the parameters a_1 to a_3 from Eq. 1 are determined for every set, then the fit can be substantially improved as indicated in Fig. 10. Current research efforts seek to generalize the dependence of the parameters a_1 to a_3 with $^{\circ}API$, R_{sb} and other field derived variables. Table 2 summarizes the statistics for these models.

Conclusions

We presented a new viscosity correlation derived from Pedersen's corresponding states model which does not require compositional information and can be used for black oil fluids. This model can be easily implemented in any reservoir simulation software, it can be easily tuned, and it provides better estimates of oil viscosity than the existing correlations.

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Figure captions

Fig. 1. - Schematics of a Differential Liberation Test on a Crude Oil

- Fig. 2. - Predicted Viscosity vs Experimental Viscosity – Khan *et al.* model (Data Set 1)
- Fig. 3. - Predicted Viscosity vs Experimental Viscosity – Petrosky model (Data Set 1)
- Fig. 4. - Predicted Viscosity vs Experimental Viscosity – Adapted Pedersen’s model (Data Set 1)
- Fig. 5. - Predicted Viscosity vs Experimental Viscosity – This work (Data Set 1)
- Fig. 6. - Predicted Viscosity vs Experimental Viscosity – Khan model (Data Set 2 –Core Labs)
- Fig. 7. - Predicted Viscosity vs Experimental Viscosity – Petrosky model (Data Set 2 – Core Labs)
- Fig. 8. - Predicted Viscosity vs Experimental Viscosity – Adapted Pedersen’s model (Data Set 2 – Core Lab)
- Fig. 9. - Predicted Viscosity vs Experimental Viscosity – This work (Data Set 2)
- Fig. 10. - Predicted Viscosity vs Experimental Viscosity – Tuned Pedersen’s Model set by set fit.

Table 1.

Range of input data. SI units and values in are indicated in parenthesis.

Dataset	N° Points.	Variable	Minimum	Maximum
#1	2343	Oil Density: lbm/ft ³ (<i>g/cm</i> ³)	35.11 (<i>0.562</i>)	57.31(<i>0.92</i>)
#2	150	Oil Density, lbm/ft ³ (<i>g/cm</i> ³)	24.31 (<i>0.389</i>)	57.50(<i>0.921</i>)
#1	2343	Oil Viscosity, cp	0.132	78.30
#2	150	Oil Viscosity, cp	0.13	68.90
#1	2343	Temperature, R (<i>K</i>)	540 (<i>300</i>)	766 (<i>425.5</i>)
#2	150	Temperature, R (<i>K</i>)	537 (<i>303.9</i>)	762 (<i>423.3</i>)
#1	2343	Pressure, psia (<i>MPa</i>)	14.7 (<i>0.1</i>)	5601.7 (<i>38.62</i>)
#2	150	Pressure, psia (<i>MPa</i>)	102.7 (<i>0.708</i>)	5434.7 (<i>37.47</i>)

Table 2.

Summary of the performance of black oil viscosity models.

Model	Number of Observations	Maximum Error, %	Minimum Error, %	Average Error, %
Khan	2343	81.6	-567	-28
Khan	150	66.1	-636	-60.8
Petrosky	2343	80.1	-214	4.9
Petrosky	150	44.8	-111	-1.4
Adapted Pedersen	2343	99.2	-384.	62
Adapted Pedersen	150	98.9	-382	54
This Work	2343	77.7	-317	0.9
This Work	150	58.7	-189	-0.7

Nomenclature

API	=	Oil gravity, ($API = 145/g_{,STC} - 135$)
Bo	=	Oil formation volume factor, (BBL/STB)
MW_m	=	Mixture molecular weight
P_{cm}	=	Mixture critical pressure (psia)
P	=	Pressure (psia)
R_s	=	Solution Gas/Oil Ratio, SCF/STB
T	=	Temperature (R)
T_{cm}	=	Mixture critical temperature (R)
V_{cm}	=	Mixture critical volume, (ft ³ /lbmol)
$g_{,100}$	=	Gas specific gravity at 100 psia and 60 °F (air=1)
$g_{,R}$	=	Oil specific gravity at reservoir conditions
r	=	Density (lb/ft ³)
a_n	=	parameter defined in Eq. (2)
a	=	parameter defined in Eq. (3)

m = Oil viscosity, cp

Subscripts

o = reference conditions, oil

c10 = n-decane.

r = reduced

c = critical

m = mixture

b = at bubble point, or normal boiling point (Eq. 8).

o,R = oil at reservoir conditions

g,100 = gas at 100 psia.

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Figures

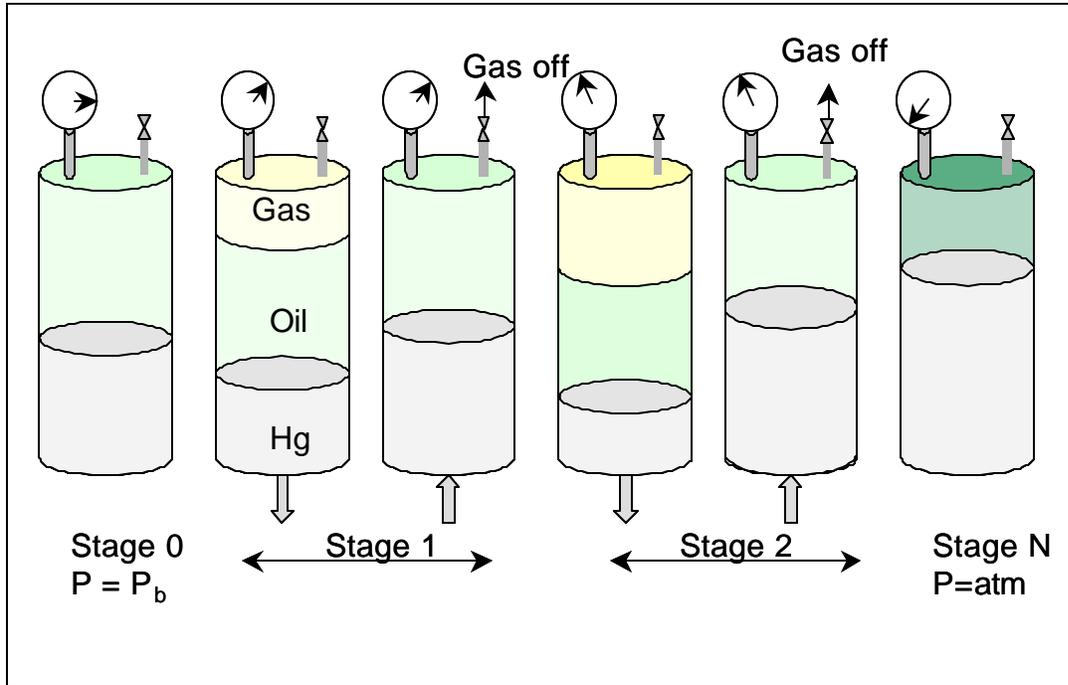
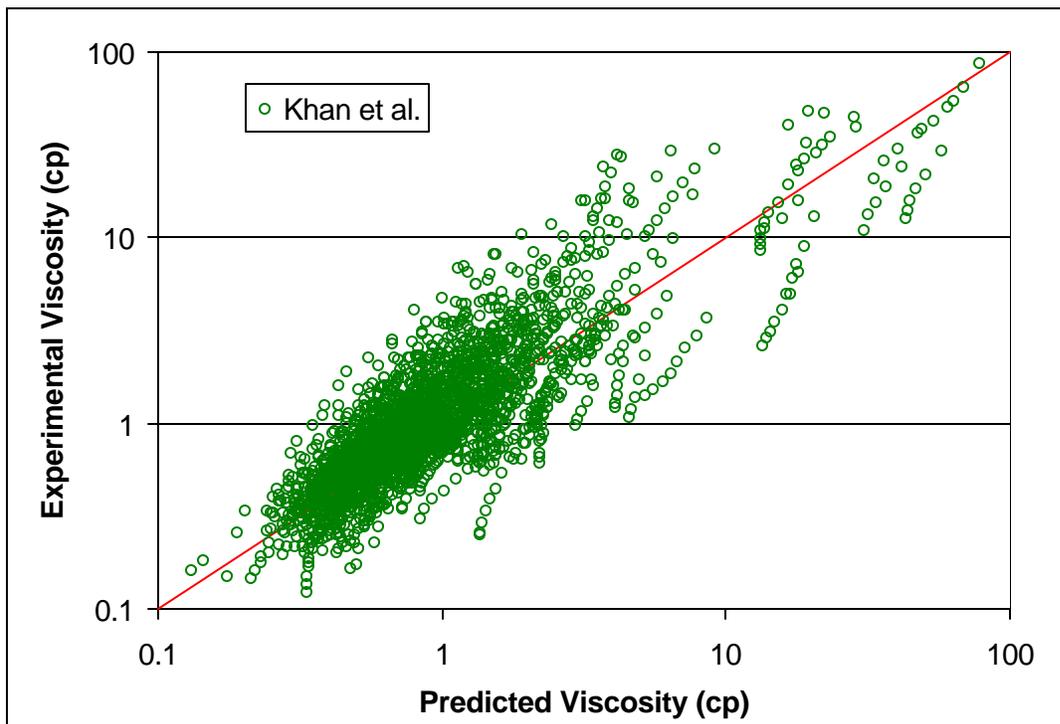


Figure 1 above – Figure 2 below



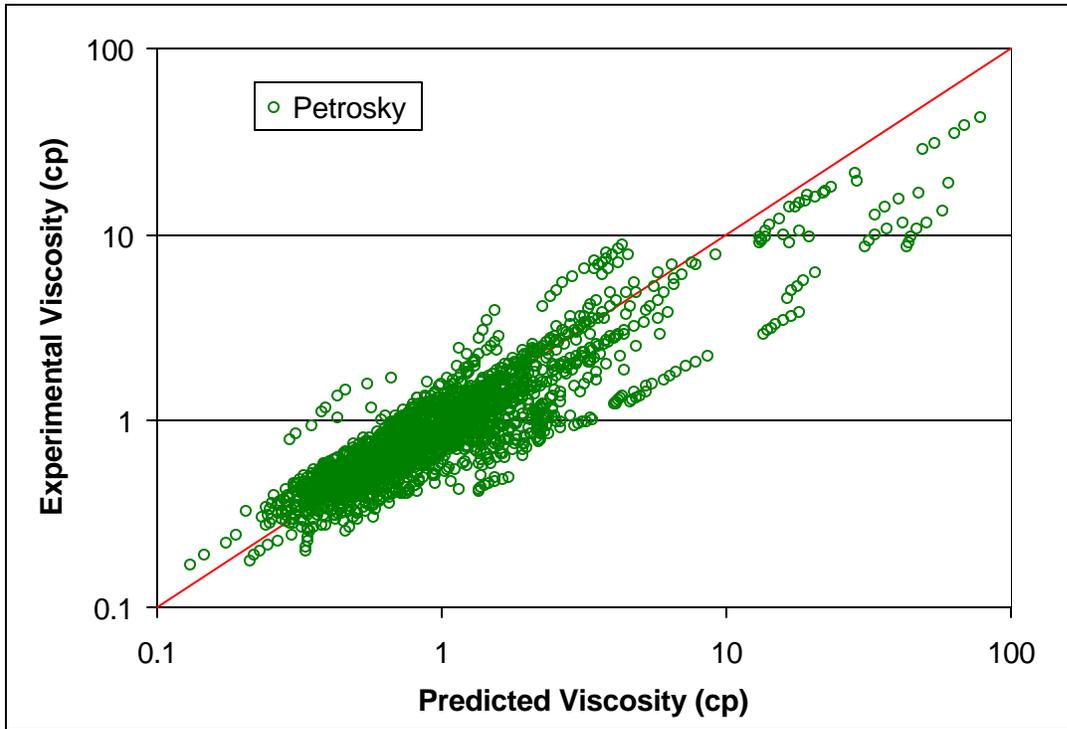
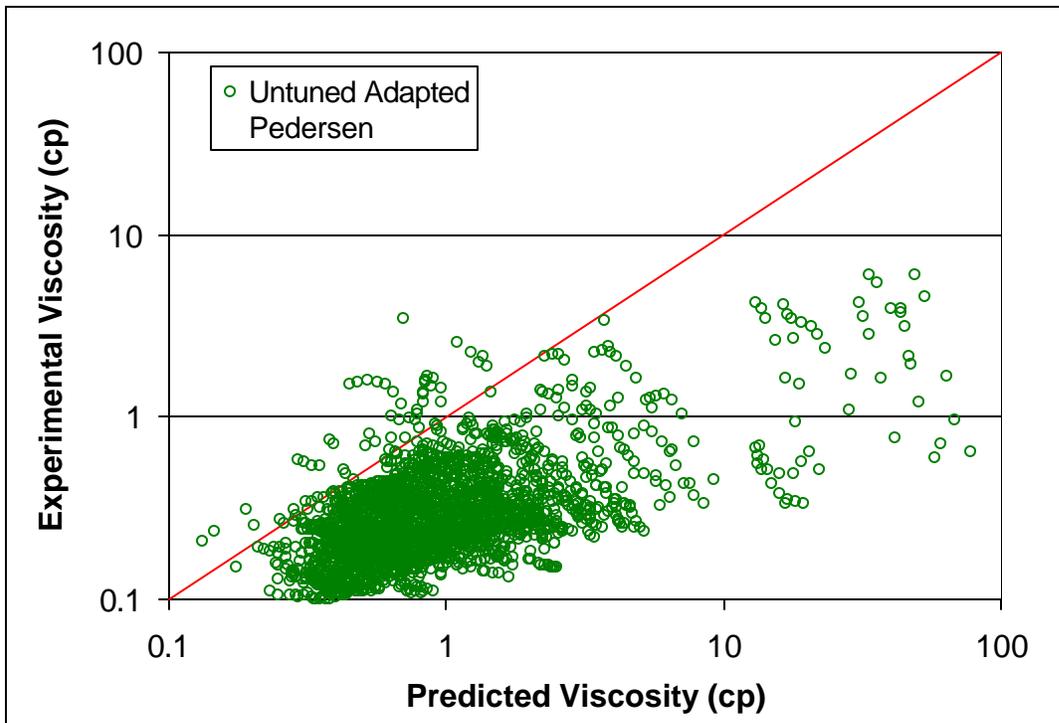


Figure 3 above – Figure 4 below



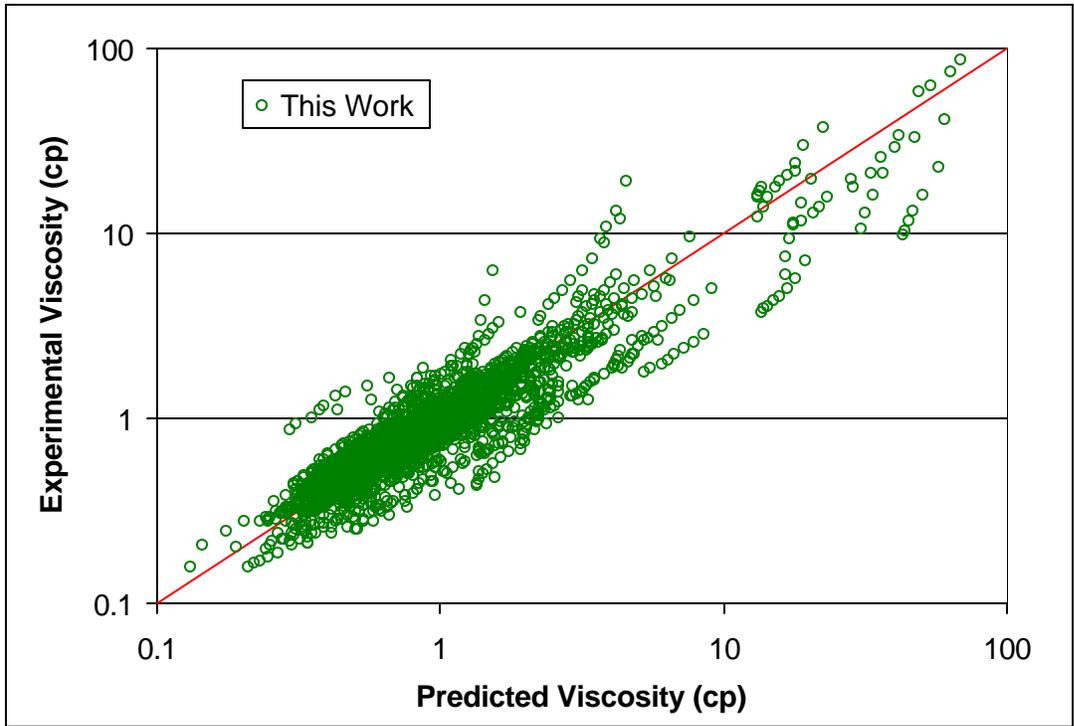
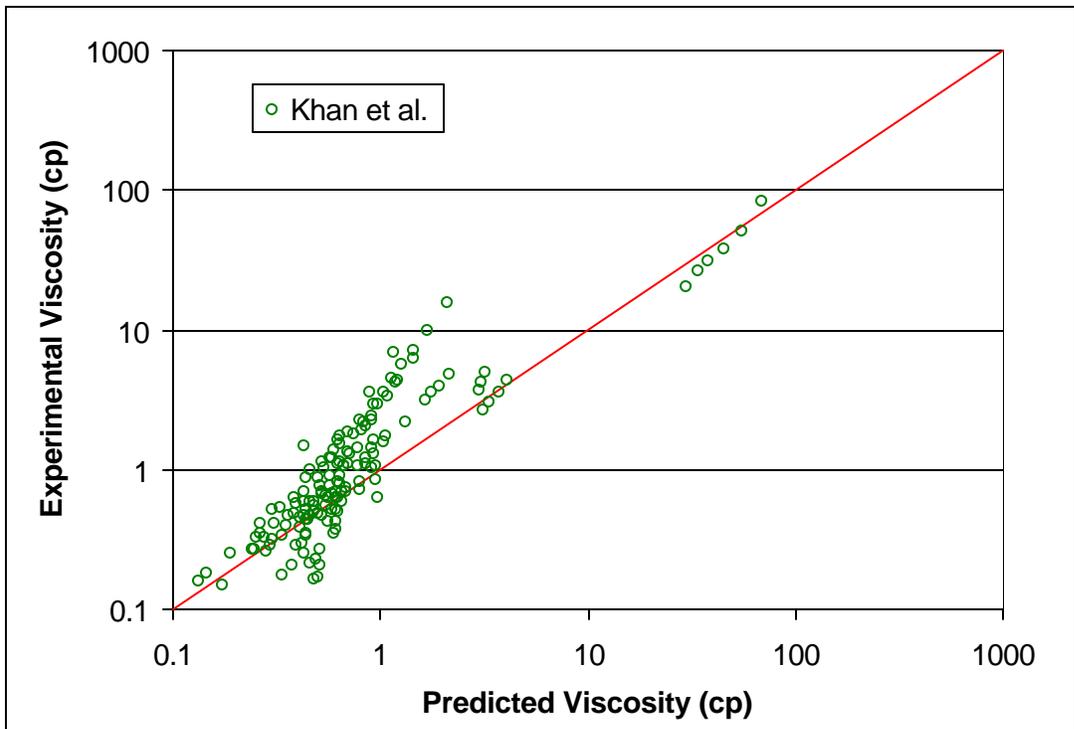


Figure 5 above – Figure 6 below



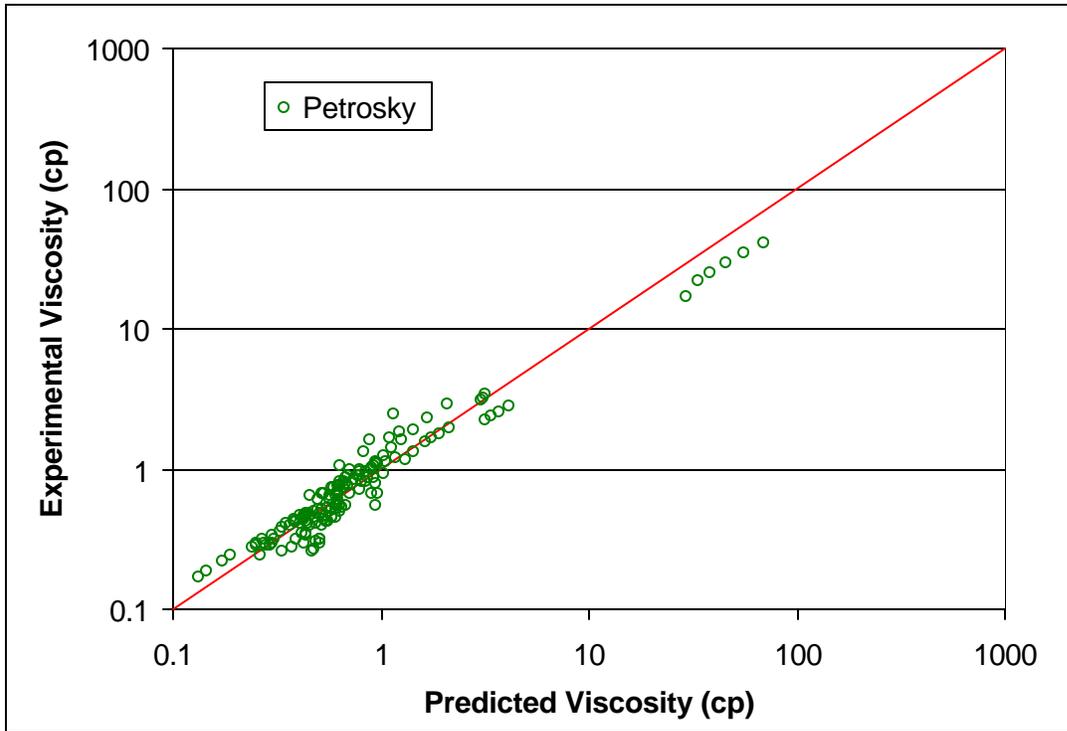
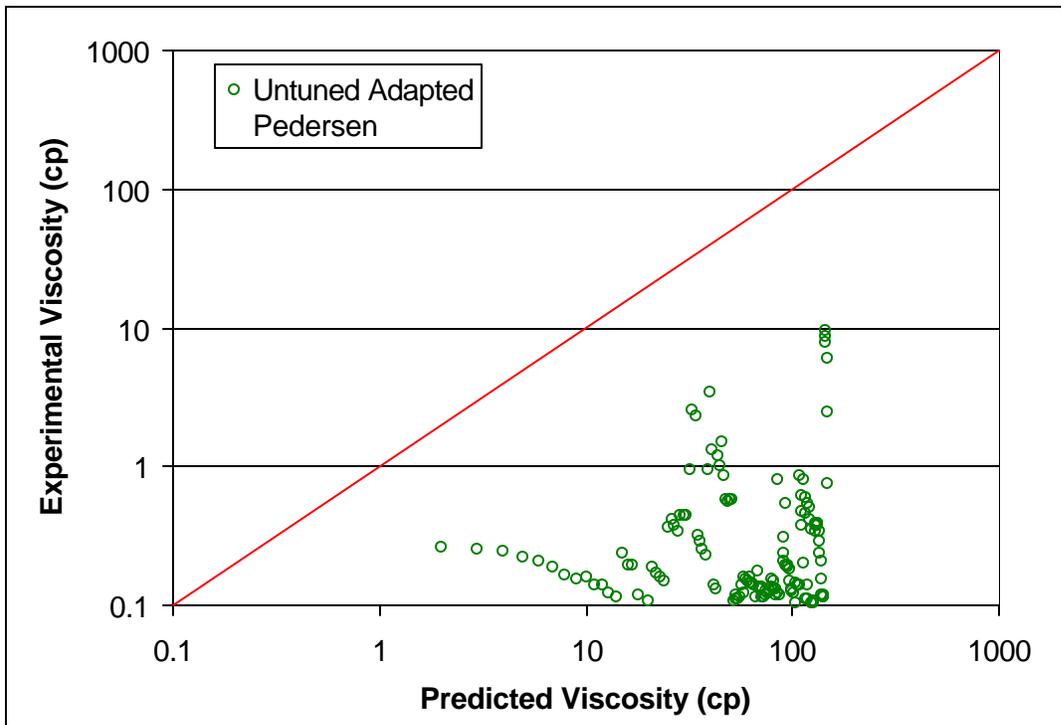


Figure 7 above – Figure 8 below



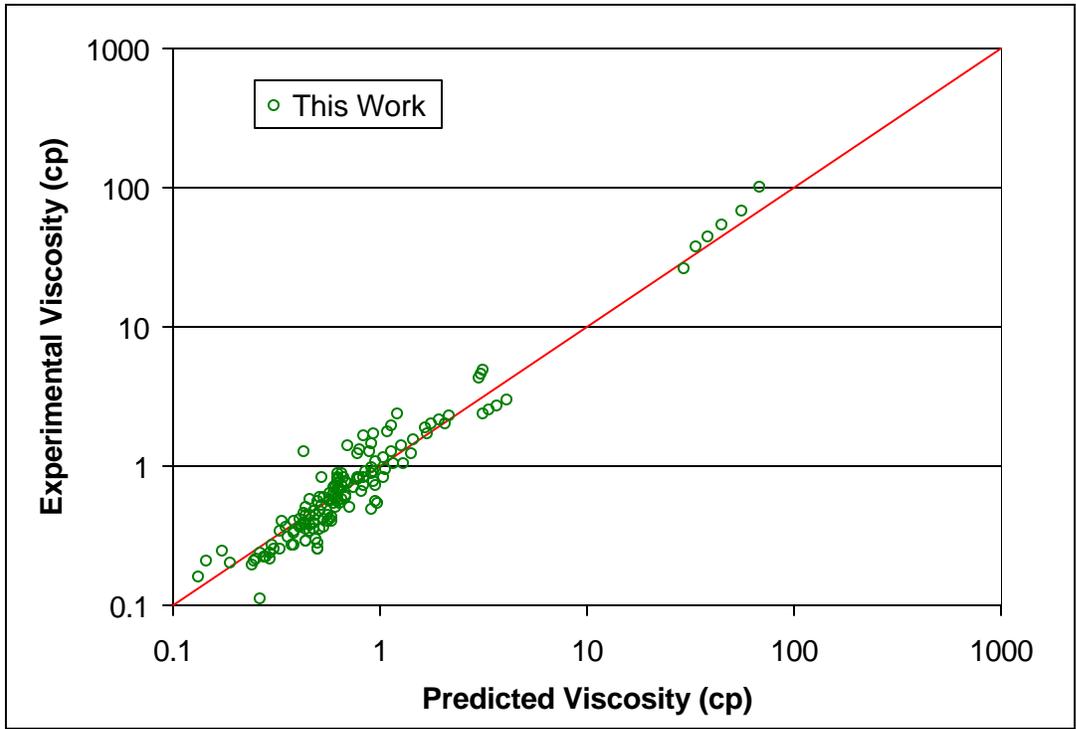


Figure 9 above- Figure 10 below

